

東海大學 97 學年度碩士班招生入學考試試題

考試科目：微積分 C 應考系所：國貿乙組

本試題共 2 頁：第 1 頁 (如有缺損或印刷不清者，應即舉手請監試人員處理)

1. Let $B(\alpha, \beta) = \int_0^1 x^\alpha (1-x)^\beta dx$, $\alpha > 0, \beta > 0$
- (a) Show that $B(\alpha, \beta) = B(\beta, \alpha)$ 5%
- (b) Show that $B(\alpha, \beta) = \frac{\alpha}{\beta+1} B(\alpha-1, \beta+1)$ 8%
2. Let $f(x) = (1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}}$
- (a) Find the Maclaurin series for f 10%
- (b) Find $f^{(4)}(0)$ and $f^{(51)}(0)$ 5%
3. (a) Let $P(t, x) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}}$, show that $\frac{\partial P}{\partial t} = \frac{1}{2} \frac{\partial^2 P}{\partial x^2}$ 10%
- (b) Suppose that f and g are functions of x and y such that
- $$\frac{\partial f}{\partial x} = \frac{\partial g}{\partial y} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$$
- and suppose $\frac{\partial f}{\partial x} = 0$, $f(1, 2) = g(1, 2) = 8$, $f(0, 0) = 4$ 10%
- Find $f(x, y)$ and $g(x, y)$.
4. For $f(x, y) = x^3 - 6x^2 - 3y^2$, $(x, y) \in [-1, 1] \times [-1, 1]$. Find the absolute maximum and absolute minimum values of f and where they occur. 17%

5. (a) Evaluate

$$\iint_R e^{x+y} dA$$

Where R is given by the inequality $|x| + |y| \leq 1$.

10%

(b) Let f be continuous on $[0,1]$ and R be the triangular region with vertices $(0,0)$, $(1,0)$ and $(0,1)$ show that

$$\iint_R f(x+y) dA = \int_0^1 uf(u) du$$

10%

6. Show that for small $|x|$ and $|y|$

$$\sqrt{\frac{1+x}{1+y}} \approx 1 + \frac{1}{2}x - \frac{1}{2}y - \frac{1}{8}x^2 + \frac{1}{4}xy + \frac{5}{8}y^2$$

15%