東海大學 98 學年度碩士班招生入學考試試題

考試科目：微積分 C　　應考系所：國貿系

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國貿系微積分 C

2009/3/30

15:40~17:10

1. Consider a population $P = P(t)$ with constant relative birth and death rate α and β respectively and a constant emigration rate m, where $\alpha > 0$, $\beta > 0$, $m > 0$ and $\alpha > \beta$, the rate of change of the population at time t is modeled by the differential equation

$$\frac{dP}{dt} = kP - m \quad \text{where} \quad k = \alpha - \beta \qquad (20\%)$$

(a) Find the solution of this equation that satisfies the initial condition $P(0) = P_0$. 10%

(b) What condition on m will lead to an exponential expansion of the population? a decline population? 4%

(c) In 1847, the population of Ireland was about 8 million and the difference between the relative birth and death rates was 1.6% of the population, because of the potato famine in the 1840s and 1850s about 210,000 inhabitants per year emigrated from Ireland, was the population expanding or declining at that time, explain it. 6%

2. Evaluate the integral (20%)

(a) $\int_0^1 \int_x^1 \sin(y^2)\,dydx$. 10%

(b) $\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\,dxdy$. 10%

3. Locate the global extrema of the function

$$f(x,y) = 3xy - x^3 - y^3 + 15, \quad -\frac{3}{2} \le x,\ y \le 2. \qquad (20\%)$$

4. Suppose x units of labor and y units of capital are required to preduce

$$f(x,y) = Kx^{\frac{1}{3}}y^{\frac{2}{3}}$$

units of certain product, where $K > 0$. Suppose now the production level is fixed at $2000K$, what amounts x of labor and y of capital minimize the cost function $200x + 300y$. (20%)

5. Find a quadratic approximation to $f(x,y) = \sin x \sin y$ near the origin, how accurate is the approximation if $|x| \le 0.1$ and $|y| \le 0.1$. (20%)