

95 學年度國際貿易研究所入學考題：統計學試題

注意：所有試題之題號及其答案皆須依序寫在答案卷上，否則不予計分。(題目共 3 頁)

一、名詞解釋 (共 4 題，每題 5 分)

1. Descriptive Statistics vs. Inferential Statistics
2. Classical Assumptions vs. BLUE
3. Dummy Variable vs. Chow Test
4. Mann-Whitney Test vs. Run Test

二、計算題 (共 11 題，每題分數皆標示於各題題目內)

1. If X is the proportion of persons who will respond to one kind of mail-order solicitation, Y is the proportion of persons who will respond to another kind of mail-order solicitation, and the joint probability density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{2}{5}(x+4y) & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that at most 50 percent will respond to the second kind of mail-order solicitation given that there has been a 20 percent response to the first kind of mail-order solicitation. (5 points)

2. The number of arrivals at a car wash is Poisson distributed with a mean of 8 cars per hour. What is the probability that 10 cars will arrive in the next two hours? (5 points)

3. You want to test the following hypothesis with $\sigma = 50$, $n = 100$, and $\alpha = 0.05$.

$$H_0: \mu = 300 \quad H_1: \mu > 300$$

Find the probability of a Type II error when the true $\mu = 310$. (5 points)

4. In deciding where to invest her retirement fund, an investor recorded the weekly returns of two portfolios for 1 year and obtained the following data.

$$\text{Portfolio 1: } s_1^2 = 0.0261, \quad n_1 = 52$$

$$\text{Portfolio 2: } s_2^2 = 0.0875, \quad n_2 = 52$$

Can we conclude at the 5% significance level that portfolio 2 is riskier than portfolio 1? (5 points)

5. To address the rising cost of absenteeism (缺席率), the personnel department of a large corporation recorded the weekdays during which individuals in a sample of 360 absentees were away over the past several months. Do these data suggest that absenteeism is higher on some days of the week than on others? ($\alpha = 0.05$) (5 points)

Day of the Week	Monday	Tuesday	Wednesday	Thursday	Friday
Number Absent	86	62	70	68	74

6. A company would like to know whether health-conscious adults are more likely to purchase its Special X cereal, and conducted a survey. Each adult was classified as either a member of the health-conscious group (code=A) or not (code=B). Each respondent was also asked whether he or she buys Special X (1=no, 2=yes). (5 points)

	Group A	Group B
Buy	32	56
Not Buy	199	563

Can we infer that health-conscious adults are more likely to buy Special X ($\alpha = 0.05$)?

7. In addition to the information in the above question, an adult's educational attainment (1=did not finish high school, 2=finished high school, 3=finished college or university, 4=post graduate degree) was recorded. Can we infer that whether a person buys Special X is related to the four educational groups and?

	Education Level				Total
	1	2	3	4	
Buy	70	376	229	87	762
Not Buy	5	42	33	8	88
Total	75	418	262	95	850

注意：本題不必計算過程，只需（1）設立虛無假設與對立假設，並（2）說明採用何種統計檢定方式。必須（1）與（2）項同時答對才給分。 (5 points)

8. Some MBA programs require referee to rank applicants under the following 4 ranks:
 Rank 1 = The candidate is in the bottom 50% of applicants; Rank 2 = The candidate is in the top 50% of applicants, but not in the top 25%; Rank 3 = The candidate is in the top 25% of applicants, but not in the top 10%; Rank 4 = In the top 10% of applicants.
 A random sample of recently graduated MBAs was drawn. For each, the rating of the referee and the MBA grade-point average (GPA) were recorded. Do these data present sufficient evidence to infer that the letter of reference and the MBA GPA are related?

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9. If the joint probability density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{1}{4}(2x + y) & \text{for } 0 < x < 1, 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the conditional variance of Y given X = 1/4. (6 points)

10. A random variable X has an exponential distribution $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$.

Derive the second central moment of X. (10 points)

11. Consider the multiple regression model: (24 points)

$$Y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + e_i$$

with the 9 observations on Y_i, X_{1i}, X_{2i} , and X_{3i} given in «Table1» Use a hand calculator to answer the following questions:

«Table1» Data for Excise

Y_i	X_{1i}	X_{2i}	X_{3i}
1	1	0	1
2	1	1	-2
3	1	2	1
-1	1	-2	0
0	1	1	-1
-1	1	-2	-1
2	1	0	1
1	1	-1	1
2	1	1	0

(a) Calculate the observations in term of deviations from their means. That is, find

$$x_{2i} = X_{2i} - \bar{X}_2 \quad x_{3i} = X_{3i} - \bar{X}_3 \quad y_i = Y_i - \bar{Y}$$

(b) Calculate $\sum y_i x_{2i}, \sum x_{2i}^2, \sum y_i x_{3i}, \sum x_{2i} x_{3i}, \sum x_{3i}^2$

(c) Find least squares estimates b_1, b_2, b_3 .

(d) Find the least squares residuals $\hat{e}_1, \hat{e}_2, \dots, \hat{e}_9$.

(e) Find the variance estimate $\hat{\sigma}^2$.

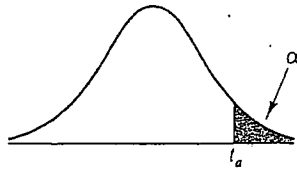
(f) Find the sample correlation between X_2 , and X_3 .

(g) Find the standard error for b_2 and t -value.

(h) Find $SSE, SST, SSR, R^2, \bar{R}^2$ and F -value.

Critical Values from the t Distribution

附表共5頁, 第2頁



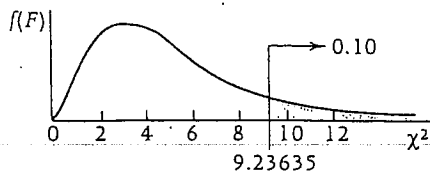
Values of α for one-tailed test and $\alpha/2$ for two-tailed test

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.894
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
40	1.303	1.684	2.021	2.423	2.704	3.307
50	1.299	1.676	2.009	2.403	2.678	3.261
60	1.296	1.671	2.000	2.390	2.660	3.232
70	1.294	1.667	1.994	2.381	2.648	3.211
80	1.292	1.664	1.990	2.374	2.639	3.195
90	1.291	1.662	1.987	2.368	2.632	3.183
100	1.290	1.660	1.984	2.364	2.626	3.174
150	1.287	1.655	1.976	2.351	2.609	3.145
200	1.286	1.653	1.972	2.345	2.601	3.131
∞	1.282	1.645	1.960	2.326	2.576	3.090

The Chi-Square Table

附表共5頁, 第3頁

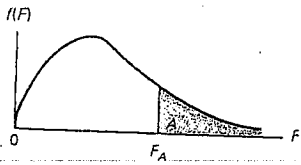
Values of χ^2 for Selected Probabilities



Example: df (Number of degrees of freedom) = 5, the tail above $\chi^2 = 9.23635$ represents 0.10 or 10% of the area under the curve.

Degrees of Freedom	Area in Upper Tail									
	.995	.99	.975	.95	.9	.1	.05	.025	.01	.005
1	0.0000393	0.0001571	0.0009821	0.0039322	0.0157907	2.7055	3.8415	5.0239	6.6349	7.8794
2	0.010025	0.020100	0.050636	0.102586	0.210721	4.6052	5.9915	7.3778	9.2104	10.5965
3	0.07172	0.11483	0.21579	0.35185	0.58438	6.2514	7.8147	9.3484	11.3449	12.8381
4	0.20698	0.29711	0.48442	0.71072	1.06362	7.7794	9.4877	11.1433	13.2767	14.8602
5	0.41175	0.55430	0.83121	1.14548	1.61031	9.2363	11.0705	12.8325	15.0863	16.7496
6	0.67573	0.87208	1.23734	1.63538	2.20413	10.6446	12.5916	14.4494	16.8119	18.5475
7	0.98925	1.23903	1.68986	2.16735	2.83311	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.34440	1.64651	2.17972	2.73263	3.48954	13.3616	15.5073	17.5345	20.0902	21.9549
9	1.73491	2.08789	2.70039	3.32512	4.16816	14.6837	16.9190	19.0228	21.6660	23.5893
10	2.15585	2.55820	3.24696	3.94030	4.86518	15.9872	18.3070	20.4832	23.2093	25.1881
11	2.60320	3.05350	3.81574	4.57481	5.57779	17.2750	19.6752	21.9200	24.7250	26.7569
12	3.07379	3.57055	4.40378	5.22603	6.30380	18.5493	21.0261	23.3367	26.2170	28.2997
13	3.56504	4.10690	5.00874	5.89186	7.04150	19.8119	22.3620	24.7356	27.6882	29.8193
14	4.07466	4.66042	5.62872	6.57063	7.78954	21.0641	23.6848	26.1189	29.1412	31.3194
15	4.60087	5.22936	6.26212	7.26093	8.54675	22.3071	24.9958	27.4884	30.5780	32.8015
16	5.14216	5.81220	6.90766	7.96164	9.31224	23.5418	26.2962	28.8453	31.9999	34.2671
17	5.69727	6.40774	7.56418	8.67175	10.08518	24.7690	27.5871	30.1910	33.4087	35.7184
18	6.26477	7.01490	8.23074	9.39045	10.86494	25.9894	28.8693	31.5264	34.8052	37.1564
19	6.84392	7.63270	8.90651	10.11701	11.65091	27.2036	30.1435	32.8523	36.1908	38.5821
20	7.43381	8.26037	9.59077	10.85080	12.44260	28.4120	31.4104	34.1696	37.5663	39.9969
21	8.03360	8.89717	10.28291	11.59132	13.23960	29.6151	32.6706	35.4789	38.9322	41.4009
22	8.64268	9.54249	10.98233	12.33801	14.04149	30.8133	33.9245	36.7807	40.2894	42.7957
23	9.26038	10.19569	11.68853	13.09051	14.84795	32.0069	35.1725	38.0756	41.6383	44.1814
24	9.88620	10.85635	12.40115	13.84842	15.65868	33.1962	36.4150	39.3641	42.9798	45.5584
25	10.51965	11.52395	13.11971	14.61140	16.47341	34.3816	37.6525	40.6465	44.3140	46.9280
26	11.16022	12.19818	13.84388	15.37916	17.29188	35.5632	38.8851	41.9231	45.6416	48.2898
27	11.80765	12.87847	14.57337	16.15139	18.11389	36.7412	40.1133	43.1945	46.9628	49.6450
28	12.46128	13.56467	15.30785	16.92788	18.93924	37.9159	41.3372	44.4608	48.2782	50.9936
29	13.12107	14.25641	16.04705	17.70838	19.76774	39.0875	42.5569	45.7223	49.5878	52.3355
30	13.78668	14.95346	16.79076	18.49267	20.59924	40.2560	43.7730	46.9792	50.8922	53.6719
40	20.70658	22.16420	24.43306	26.50930	29.05052	51.8050	55.7585	59.3417	63.6908	66.7660
50	27.99082	29.70673	32.35738	34.76424	37.68864	63.1671	67.5048	71.4202	76.1538	79.4898
60	35.53440	37.48480	40.48171	43.18797	46.45888	74.3970	79.0820	83.2977	88.3794	91.9518
70	43.27531	45.44170	48.75754	51.73926	55.32894	85.5270	90.5313	95.0231	100.4251	104.2148
80	51.17193	53.53998	57.15315	60.39146	64.27784	96.5782	101.8795	106.6285	112.3288	116.3209
90	59.19633	61.75402	65.64659	69.12602	73.29108	107.5650	113.1452	118.1359	124.1162	128.2987
100	67.32753	70.06500	74.22188	77.92944	82.35813	118.4980	124.3421	129.5613	135.8069	140.1697

Table 6(a) Critical Values of $F: A = .05$



ν_2	ν_1	NUMERATOR DEGREES OF FREEDOM								
		1	2	3	4	5	6	7	8	9
1		161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2		18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3		10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4		7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5		6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6		5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7		5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8		5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9		5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10		4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11		4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12		4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13		4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14		4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15		4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16		4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17		4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18		4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19		4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20		4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21		4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22		4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23		4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24		4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25		4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26		4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27		4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28		4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29		4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30		4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40		4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60		4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120		3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞		3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

SOURCE: From M. Merrington and C. M. Thompson, "Tables of Percentage Points of the Inverted Beta (F)-Distribution," *Biometrika* 33 (1943): 73-88. Reproduced by permission of the Biometrika Trustees.

附表共5頁,第5頁

Table 6(a)
continued

ν_2	NUMERATOR DEGREES OF FREEDOM									
	10	12	15	20	24	30	40	60	120	∞
1	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

DENOMINATOR DEGREES OF FREEDOM