‘Identifying Turning Points and Business Cycles in Taiwan: A Multivariate Dynamic Markov-Switching Factor Model Approach,’

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2 authors:

Shyh-Wei Chen
Tunghai University

Jin-Lung Lin
National Dong Hwa University

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IDENTIFYING TURNING POINTS AND BUSINESS CYCLES IN TAIWAN: A MULTIVARIATE DYNAMIC MARKOV−SWITCHING FACTOR MODEL APPROACH

Shyh-Wei Chen Jin-Lung Lin*

ABSTRACT

This paper builds upon the ideas proposed by Diebold and Rudebusch (1996) and estimates a multivariate dynamic Markov–switching factor model for a vector of macroeconomic variables. The approach captures both the idea of the business cycle as expressing co–movement in several macroeconomic variables as well as the asymmetric nature of business cycle phases. We transform the empirical models into state–space representation, and adopt Kim's (1994) algorithm to implement the estimation. The empirical results suggest that the business chronologies identified by the multivariate Markov–switching factor model in terms of GDP, consumption and investment are more consistent with the CEPD–defined chronologies than those defined by the univariate Markov–switching models, especially for the post–1990 period.

Keywords: Markov–switching factor model, State–space model, Kalman filter, Business cycle

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* Shyh–Wei Chen is Assistant Professor, Department of Economics, Tunghai University. Jin–Lung Lin is Research Fellow, Institute of Economics, Academia Sinica. We would like to thank two anonymous referees and an associate editor for helpful comments and suggestions. Thanks are also due to Chang–Jin Kim for providing us with the GAUSS code used in this paper. Any remaining errors are entirely our responsibility.

1. INTRODUCTION

Following on from the initial work by Burns and Mitchell (1946), a great deal of effort has been focused on measuring business cycles and identifying their turning points. As noted by Burns and Mitchell (1946, p.3), business cycles "consist of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions and revivals..." That is, they established two defining characteristics of the business cycle. The first was the co–movement among economic variables through the cycle which was stressed by Lucas (1976). Lucas (1976) drew attention to a key business cycle fact: outputs of broadly–defined sectors move together. Lucas's view is part of a long tradition that has stressed the coordination of activity among various economic sectors and the resulting co–movement in sectoral outputs. The second characteristic was the presence of nonlinearity in the evolution of the business cycle, that is, regime switching at the turning points of the business cycle. However, as noted by Diebold and Rudebusch (1996), these two aspects of the business cycle have generally been considered in isolation from one another in the literature. For instance, Hamilton's (1989) regime–switching model characterizes nonlinearity in an individual economic variable, while the Stock and Watson (1989, 1991, 1993) linear dynamic factor model appears to accurately capture the co–movement among economic variables. Diebold and Rudebusch (1996) proposed a multivariate dynamic factor model with regime switching which encompasses these two features of the business cycle. However, they did not actually undertake estimation as maximizing the exact log–likelihood of this model involves a substantial computational burden. Recently, Kim and Yoo (1995), Chauvet (1998) and Kim and Nelson (1998) employed the idea put forward by Diebold and Rudebusch (1996) to investigate United States business cycle behavior in terms of coincident indicators, and resolved the estimation problem by approximating the maximum likelihood method and the Gibbs sampler approach, respectively.[1]

[1] For a theoretical review of business cycle theory, readers may refer to Gordon (1986), Mullineux et al. (1993) and papers refereed to therein for details.

Although there is an abundance of studies on the business cycle,[2] studies on business cycles in Taiwan are not common in the literature. Lin and Huang (1993) used the dynamic factor model, developed by Stock and Watson (1989, 1991, 1993), to measure Taiwan's leading and coincident economic indicators and their relationship with the business cycle. Huang, Kuan and Lin (1998) first applied the univariate Markov–switching model to identify the turning points of the business cycle in Taiwan. Lin and Chen (1999) and Chen and Lin (2000) also used the bivariate and time–varying Markov–switching models, respectively, to investigate the ability of the leading and coincident indicators to date Taiwan's business cycle. They found that the posterior probability of the Markov–switching model exhibited a low correlation with the business chronologies defined by the Council for Economic Planning and Development (CEPD), especially for the post–1990 period. Another related empirical study on Taiwan's business cycle is that of Huang (1999), who estimated Hamilton's model and used three kinds of regime to investigate whether the asymmetrical behavior of the "plucking model" proposed by Friedman (1969, 1993) held in Taiwan's business cycle. These studies on Taiwan's business fluctuation all share a common shortcoming: they failed to identify the business chronologies with the CEPD-defined chronologies, especially for the post–1990s period. The failure of these models can be explained by the fact that the annual growth rate of GDP from 1962 to 1998 has changing trend. The average economic growth rate over this period is 8.20 percent, and there are 40 quarters where the economic growth rate is over 11 percent. However, it is well–known that Taiwan's economic growth rates are relatively high in the periods of the 1970s and the 1980s, but slowed down for the post–1990s. The average economic growth rates for the pre–1990s and the post–1990s are 8.91 and 6.00 percent, respectively. If we estimate Hamilton's (1989) Markov–switching model with mean–switching on the GDP growth rate, the estimates of high–growth and low–growth states are 11.10 percent and 7.03 percent in Taiwan, respectively. It is reasonable that the post–1990s will be identified as a contraction phase, although there are two more CEPD–defined contraction chronologies for the post–1990s.

[2] See, for instance, the special issue of *Journal of Business and Economic Statistics, 12(3), 1994*, for various extensions of Hamilton's (1989) original model on U.S. business fluctuations.

As pointed out by Diebold and Rudebusch (1996), none of these papers simultaneously considered co–movements among economic variables and nonlinearity. This paper intends to fill this gap by applying a multivariate dynamic Markov–switching factor model to estimate and measure the business cycle and identify turning points in Taiwan. We first transform the empirical models to the state–space representation, and adopt Kim's (1994) algorithm to implement the estimation. Intuitively, Kim's (1994) algorithm is synthesis of the Hamilton filter and the Kalman filter. Our empirical results suggest that the business chronologies identified by the multivariate Markov–switching factor model in terms of GDP, consumption and investment are more consistent with the CEPD–defined chronologies than those by the univariate Markov–switching models, especially for the post–1990s.

In addition to this introduction, the rest of this paper is organized as follows. Section 2 describes both the univariate Markov–switching and multivariate Markov–switching factor models in detail. The empirical results and implications are presented in Section 3. We conclude our paper in Section 4 and discuss Kim's (1994) method in the appendix.

2. MODEL SPECIFICATION

2.1 Univariate Markov–Switching Models

Let $\tilde{y}_t$ denote the logarithmic transformation of real GDP_t (CP_t, IFIX_t or EX_t) and y_t the corresponding annual growth rate of $\tilde{y}_t$, i.e., $\tilde{y}_t = \log(\text{GDP}_t)$ and $y_t = \tilde{y}_t - \tilde{y}_{t-4}$.[3] The univariate Markov–switching model of GDP is as follows:

$$y_t = n_t + z_t \tag{1}$$

$$n_t = \beta_{S_t} \tag{2}$$

$$\beta_{S_t} = \beta_0 + \beta_1 S_t, \;\; S_t = 0, 1 \tag{3}$$

$$z_t = \phi_1 z_{t-1} + \cdots + \phi_q z_{t-q} + \varepsilon_t, \;\; \varepsilon_t \sim \text{i.i.d.N}(0, \sigma^2) \tag{4}$$

[3] The detailed definitions of the empirical variables are explained in Section 3.

where the unobserved state variable S_t follows a first order Markov chain:

$$\begin{aligned} \Pr[S_t=0|S_{t-1}=0] &= q, \quad \Pr[S_t=1|S_{t-1}=1]=p, \\ \Pr[S_t=1|S_{t-1}=0] &= 1-q, \; \Pr[S_t=0|S_{t-1}=1]=1-p \end{aligned} \tag{5}$$

We consider an unobserved latent variable S_t which takes on the value 1 when GDP is in expansion and 0 when GDP is in contraction.

2.2 A Multivariate Dynamic Factor Model with Regime–Switching

A vector of macroeconomic variables displaying co–movements with aggregate economic conditions is modeled as being composed of two stochastic autoregressive processes — a single unobserved component, which corresponds to the common factor among the observable variables, and idiosyncratic components.[4] A stochastic trend is not included in the dynamic factor model based on evidence that each of the series studied might be integrated but not cointegrated.[5] The univariate unobserved component model is extended to become a dynamic Markov–switching factor model, i.e., MS(s)–DF(q, k), as follows:

$$\mathbf{y}_t=\boldsymbol{\gamma}(B)n_t+\mathbf{z}_t \tag{6}$$

$$\phi(B)n_t=\beta_{S_t}+\eta_t, \;\; \eta_t\sim \text{i.i.d.N}(0,1) \tag{7}$$

$$\beta_{S_t}=\beta_0+\beta_1 S_t, \;\; S_t=0,1 \tag{8}$$

$$\mathbf{z}_t=\mathbf{A}_1\mathbf{z}_{t-1}+\cdots+\mathbf{A}_k\mathbf{z}_{t-k}+\boldsymbol{\varepsilon}_t, \;\; \boldsymbol{\varepsilon}_t\sim \text{MVN}(\mathbf{0},\boldsymbol{\Sigma}) \tag{9}$$

where $\mathbf{y}_t=[y_{1t},y_{2t},\cdots,y_{nt}]'$ is an $(N\times 1)$ vector of the annual growth rate of macroeconomic variables and its deviations from the their means and $\mathbf{Z}_t=[z_{1t},z_{2t},\cdots,z_{nt}]'$ is an $(N\times 1)$ vector stationary series. The processes of $\mathbf{y}_t$ are driven by an unobserved

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[4] The common factor of this model will not represent the common stochastic trend component of the variables, defined as in Stock and Watson (1988). The common factor in this model is to capture the business cycle features among the macroeconomic variables.

[5] The evidence of a no cointegration relationship among the variables is presented in Section 3.

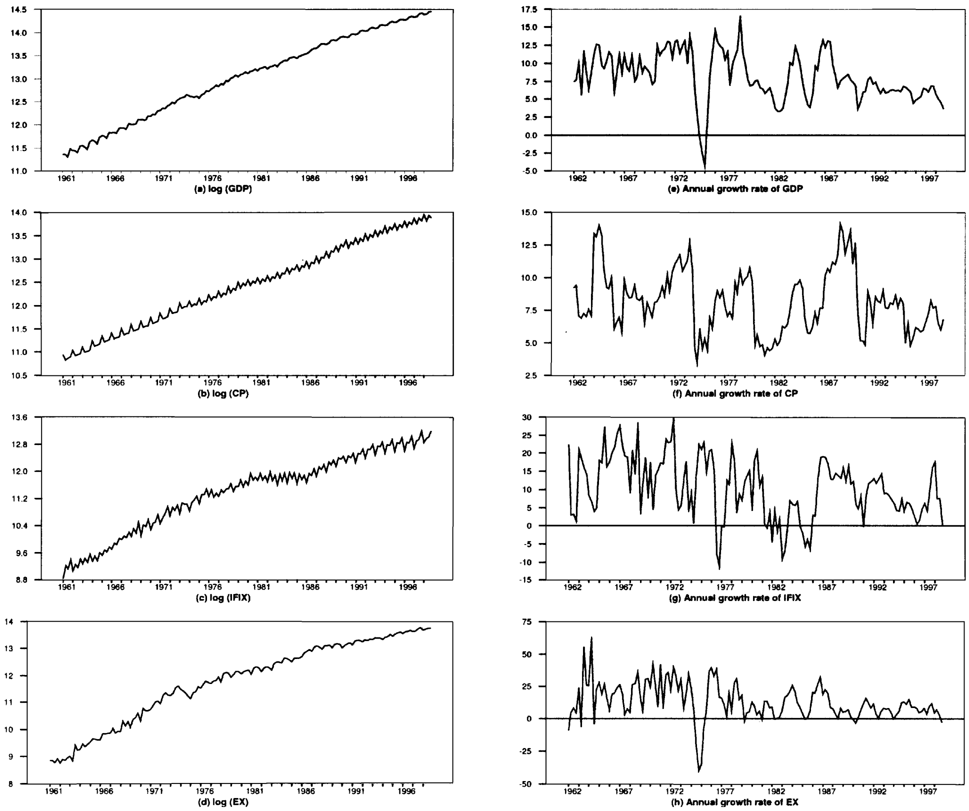

Figure 1 The Left Panels Show the Scatter Plots of GDP, CP, IFIX and EX (in log form), and Their Corresponding Annual Growth Rates Are in the Right Panels

component for all variables denoted by n_t and by an idiosyncratic component given by $\mathbf{z}_t$. $\phi(B) = (1 - \phi_1 B - \cdots - \phi_q B^q)$ is a scalar lag polynomial and $\gamma(B)$ is a vector polynomial. The identification conditions of equations (6) to (9) are as follows. First, it is necessary to set the factor variance to one or give it a scale using the same units as one of their regression coefficients. For the case in which only the mean switches, normalization can be achieved by setting $\sigma^2_{\eta_t} = 1$. This is a normalization with no substantive implications. Second, the innovations in ε_t are mutually and serially uncorrelated white noise, i.e., the covariance matrix $\sum$ and $(N \times N)$ coefficient matrix $\mathbf{A}_i$, $i = 1, 2, \cdots, k$, are diagonal, to ensure that common factor and idiosyncratic terms are mutually uncorrelated at all leads and lags. In

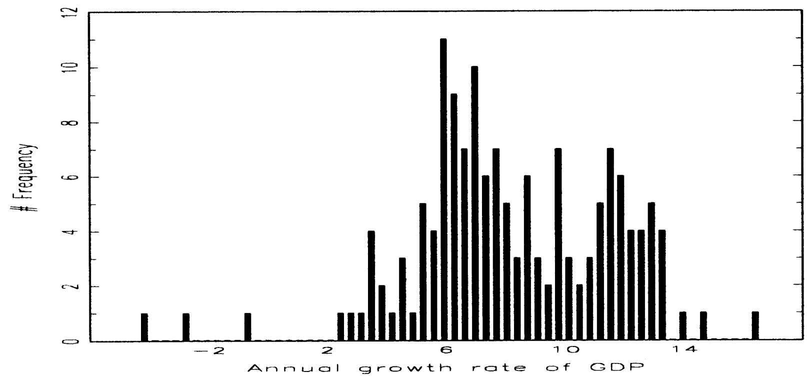

Figure 2 The Histogram for the Annual Growth Rate of GDP

other words, the model is complete, i.e., $\mathbf{y}_t$ is determined only from contemporaneous and past values of n_t and $\mathbf{z}_t$ and, since there is only one factor, n_t is the only source of co–movements of the n time series $\mathbf{y}_t$. Additionally, the random variables $\mathbf{y}_t$, $\mathbf{z}_t$ and ε_t are assumed to be covariance stationary. For a more detailed discussion of the identification conditions of dynamic factor models, readers are referred to Geweke and Singleton (1981), Stock and Watson (1989, 1991) and Chauvet (1998).

3. EMPIRICAL RESULTS

3.1 Data Analysis

According to Mullineux et al. (1993, p.1), "a very broad view of the meaning of the term 'business cycle', namely: the existence of (negative) serial correlation in key real macroeconomic aggregate series, such as output, consumption and unemployment, and nominal series, such as inflation, and co–movements between various series; in the sense that there are leading or lagging (as well as coincident) relationships between them over time, which may or may not imply causality." Our empirical variables consist of real gross domestic product (GDP), real private consumption expenditure (CP), real gross domestic fixed capital formation (IFIX) and real exports of goods and services (EX). [6] The seasonally–unadjusted real GDP

[6] See Zarnowitz and Moore (1986) who also emphasized the cyclical conformity and coherence of numerous macroeconomic variables, which implies that these series are driven by a common stochastic component.

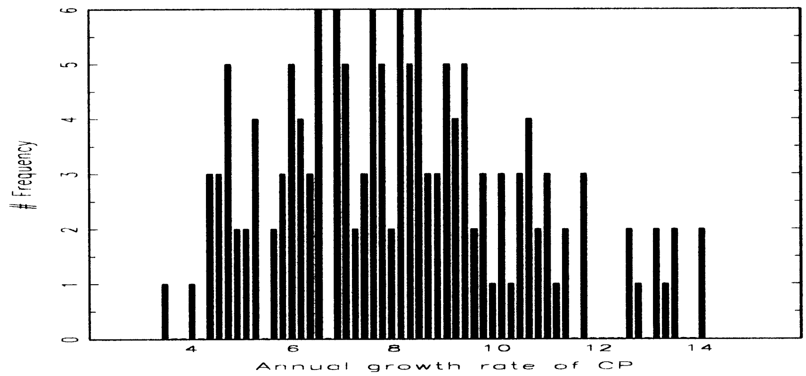

Figure 3 The Histogram for the Annual Growth Rate of CP

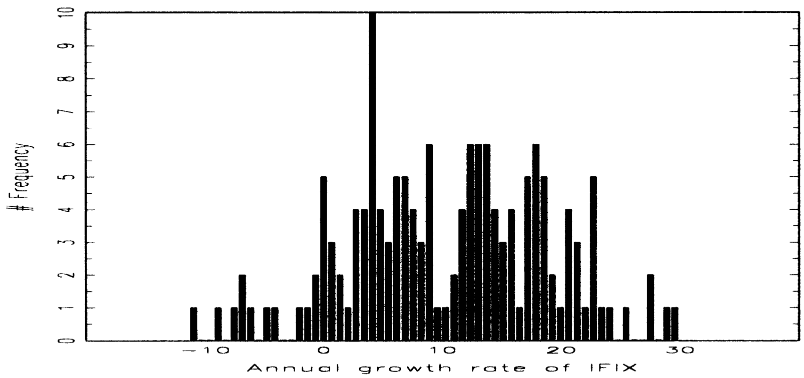

Figure 4 The Histogram for the Annual Growth Rate of IFIX

is used as a major summative measure of the business cycle, and all the series are taken from the AREMOS data bank for the Taiwan Area. The empirical data comprise quarterly data from 1961:Q1 to 1998:Q4, which amount to 152 observations. The reason we use seasonally–unadjusted data is as follows. The simulation results of Franses and Paap (1999) showed that for the seasonally–adjusted data, based on the typical two–sided moving average filter as in X–11 filters, the business cycle fluctuations tend to be smoothed out. Lin and Chen (1999) also found similar results for Taiwan data. The scatter plots of the series after logarithmic transformation and their corresponding annual growth rates are included in Figure 1. It is obvious that there is an upward trend and also seasonality in our empirical series. The seasonal

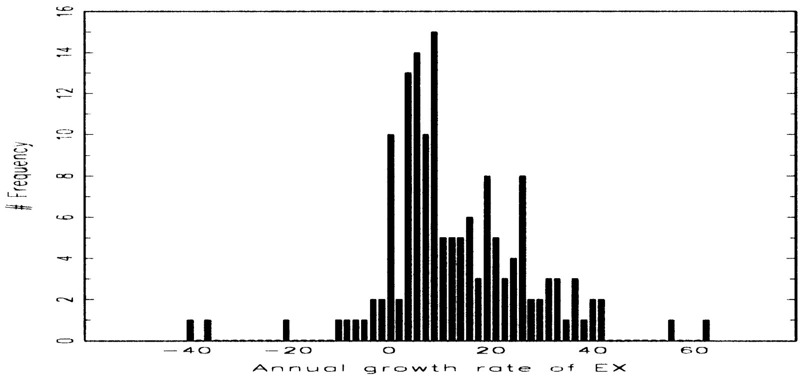

Figure 5 The Histogram for the Annual Growth Rate of EX

unit root test procedure of Hylleberg et al. (1990) is adopted to detect the seasonality of each series. The test is to perform the following regression:

$$\Phi_q^*(B)y_{4,t} = \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \varepsilon_t \tag{10}$$

where $\Phi_q^*(B) = 1 - a_1 B - a_2 B^2 - \ldots - a_q B^q$ with $q = 4$, $y_{1,t} = (1 + B + B^2 + B^3)\tilde{y}_t$, $y_{2,t} = -(1 - B + B^2 - B^3)\tilde{y}_t$, $y_{3,t} = -(1 - B^2)\tilde{y}_t$ and $y_{4,t} = (1 - B^4)\tilde{y}_t$, $\tilde{y}_t$ is the logarithmic transformation of the raw data. For testing the root 1 (zero frequency) and root -1 (2 cycles per year), this is simply a test of $\pi_1 = 0$ and $\pi_2 = 0$, respectively. For the complex root, it is equivalent to testing $\pi_3 = 0$ and $\pi_4 = 0$, or a joint test $\pi_3 \cap \pi_4 = 0$. The results are summarized in Table 1. The table shows that the seasonally–unadjusted GDP, CP and IFIX appear to have a seasonal unit root, and that EX has only a regular unit root.[7] From Observing the histogram of annual growth rates for each series as shown in Figures 2 to 5, it is apparent that each series may not come from a unimodal distribution. This is especially the case for the histogram of GDP, which has an obvious bimodal appearance. In the univariate analysis, data are expressed as one hundred times the fourth difference of the logarithm of each series. In the multivariate case, data are standardized by subtracting the sample mean from each variable and dividing by the corresponding standard deviation.

[7] We also performed the augmented Dickey–Fuller τ_τ test for the presence of unit root for each series, and it was found to be unable to reject the null hypothesis of integration against the alternative of stationarity at the 5% significant level.

Table 1 Seasonal Unit Root Test Results

GDP	Lags	$\pi_1 = 0$	$\pi_2 = 0$	$\pi_3 = 0$	$\pi_4 = 0$	$\pi_3 \cap \pi_4 = 0$
None	4	4.441	−2.095	−1.524	−2.043	3.238
I only	4	−3.351	−2.041	−1.551	−1.839	2.882
I,SD	4	−3.342	−1.891	−1.349	−2.401	3.794
I,Tr	4	−0.466	−2.030	−1.550	−1.827	2.860
I,SD,Tr	4	−0.398	−1.881	−1.348	−2.385	3.758
CP	Lags	$\pi_1 = 0$	$\pi_2 = 0$	$\pi_3 = 0$	$\pi_4 = 0$	$\pi_3 \cap \pi_4 = 0$
None	4	3.679	0.646	−1.434	−0.860	1.403
I only	4	−1.254	0.636	−1.473	−0.791	1.404
I,SD	4	−1.221	−2.076	−1.063	−1.600	1.868
I,Tr	4	−2.697	0.611	−1.517	−0.725	1.420
I,SD,Tr	4	−2.647	−2.046	−1.198	−1.526	1.909
IFIX	Lags	$\pi_1 = 0$	$\pi_2 = 0$	$\pi_3 = 0$	$\pi_4 = 0$	$\pi_3 \cap \pi_4 = 0$
None	4	3.052	−1.092	−1.137	−1.799	2.295
I only	4	−3.006	−1.018	−0.965	−1.777	2.068
I,SD	4	−2.739	−4.133	−1.420	−2.162	3.426
I,Tr	4	−2.045	−1.011	−1.007	−1.764	2.087
I,SD,Tr	4	−2.008	−4.118	−1.478	−2.144	3.476
EX	Lags	$\pi_1 = 0$	$\pi_2 = 0$	$\pi_3 = 0$	$\pi_4 = 0$	$\pi_3 \cap \pi_4 = 0$
None	4	4.020	−2.779	−3.246	−1.840	7.230
I only	4	−4.092	−2.547	−3.043	−1.447	5.825
I,SD	4	−3.751	−3.638	−5.271	−1.989	16.795
I,Tr	4	−1.291	−2.542	−3.044	−1.443	5.822
I,SD,Tr	4	−1.419	−3.633	−5.279	−1.985	16.827
Critical Value (5%)		$\pi_1 = 0$	$\pi_2 = 0$	$\pi_3 = 0$	$\pi_4 = 0$	$\pi_3 \cap \pi_4 = 0$
None		−1.94	−1.95	−1.92	−1.65	3.16
I only		−2.87	−1.92	−1.90	−1.66	3.12
I,SD		−2.91	−2.89	−3.38	−1.96	6.61
I,Tr		−3.44	−1.95	−1.92	−1.66	3.07
I,SD,Tr		−3.49	−2.91	−3.41	−1.92	6.57

Note: I=Intercept. SD=Seasonal Dummy. Tr=Trend.

Table 2 Estimates of Univariate Markov-Switching Models

	GDP	CP	IFIX	EX
ϕ_1	1.128	0.811	0.487	0.534
	(0.080)	(0.056)	(0.140)	(0.083)
ϕ_2	−0.318		0.506	0.335
	(0.045)		(0.140)	(0.089)
σ^2	1.363	1.038	4.273	7.761
	(0.089)	(0.063)	(0.334)	(0.463)
β_0	6.506	6.200	3.066	9.722
	(0.672)	(0.510)	(1.686)	(0.611)
β_1	4.545	4.027	12.365	22.814
	(0.454)	(0.363)	(1.364)	(3.844)
p	0.850	0.926	0.881	0.909
	(0.058)	(0.031)	(0.050)	(0.063)
q	0.918	0.933	0.825	0.985
	(0.032)	(0.031)	(0.061)	(0.012)
$(1-p)^{-1}$	6.667	13.514	8.403	10.989
$(1-q)^{-1}$	12.195	14.925	5.714	66.667
# of parameters	7	6	7	7
Log–Likelihood	−299.907	−251.262	−482.262	−539.213
LR	16.760	28.012	11.094	61.186

Note: The numbers in parentheses are approximate standard errors.

3.2 Estimates of the Univariate Markov-Switching Model

The univariate Markov–switching models are equations (1) to (5). The Kalman filter iterations start with $\boldsymbol{\xi}_{0|0}^{(j)}$ equal to the unconditional expectation of the state vector and with $\mathbf{P}_{0|0}^{(j)}$ as its unconditional covariance matrix. A numerical estimation of the unknown parameters was performed using the OPTIMUM module of GAUSS 3.2 with a combination of the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm. We impose no constraints on any of the transition probabilities p and q other than the conditions that $0 \leq p \leq 1$, $0 \leq q \leq 1$ and that σ_{ε_i}, $i = 1, 2, 3$ is constrained to be positive. Several different specifications for each of the series were estimated, including the AR(1), AR(2) and AR(3) processes. The finally chosen model is based on the likelihood ratio statistic of the AR(k) with respect to the alternative of the additional autoregressive coefficient, i.e., the AR($k+1$), at the 5 percent level of significance. The estimates of the univariate Markov–switching models are given in Table 2.[8]

The first question we ask is whether each series dynamics is well–characterized by a Markov–switching model. The estimates of the mean growth rate of contraction and expansion , i.e., β_0 and $\beta_0+\beta_1$, are statistically significantly different from each other in each series. For example, the estimates are $\beta_0 = 6.506$ and $\beta_1 = 4.545$ for the series GDP, and $\beta_0 = 6.200$ and $\beta_1 = 4.027$ for the series CP, respectively. The estimation results suggest dichotomizing the economy into high–growth and low–growth states.[9] The LR at the bottom of Table 2 is the likelihood ratio test results of the following restriction

$$H_0 : \beta_1 = 0 \tag{11}$$

Under the null hypothesis, the two–regime Markov–switching model will reduce to a single–regime autoregressive model. From the results in Table 2, the traditional likelihood ratio statistic will reject the null hypothesis in favor of our nonlinear Markov–switching specification. However, these test statistics do not have the standard asymptotic distribution. The problem arises from two sources: First, under the null hypothesis, some parameters are not identified and the scores are identically zero (see Hansen (1992, 1996) for a detailed explanation of these problems). Hansen (1992, 1996) had proposed a bound test that addressed these problems, but its computational difficulty has limited its applicability. Garcia (1998) showed that the asymptotic distribution theory used for testing in the presence of such a problem appears to work also for Markov–switching models, even though its validity can be questioned because of the identical zero scores under the null estimates. Using the asymptotic distribution critical values simulated by Garcia (1998), the approximate

[8] Our model includes two lagged dependent variables, whereas Hamilton's (1989) algorithm begins with the third observation. In the state–space representation, as noted by Kim (1994), no lagged dependent variable appears. Our algorithm begins with the first observation.

[9] It should be noted that the average growth rate of GDP over the whole sample period is 8.21%, and that, except for three quarters, 1974:Q3 to 1975:Q1, the growth rates of GDP were well above zero in Taiwan. Hence, contraction here means low growth rather than recession.

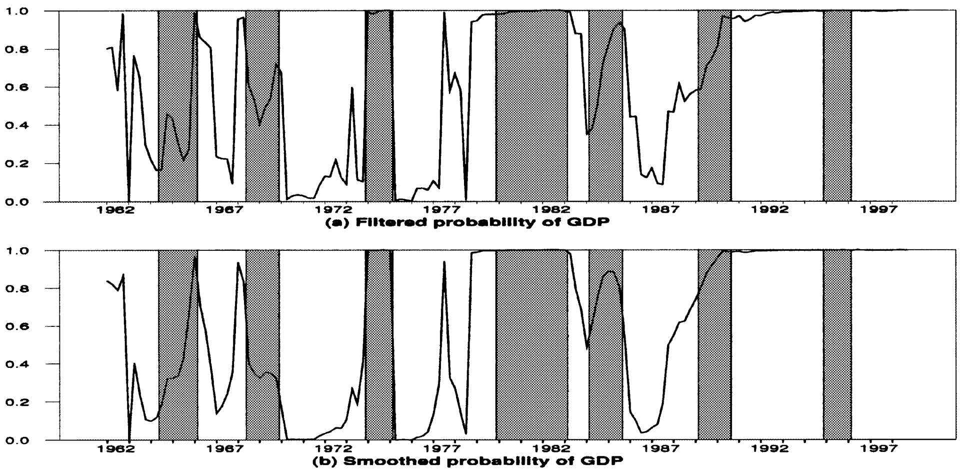

Figure 6 Filtered and Smoothed Probabilities of GDP. The Shaded Areas Are the Contraction Periods Determined by the CEPD

critical value is 8.59 (12.24) at the 5% (1%) significance level. It is obvious that our LR results still reject the autoregressive model under the null hypothesis and are in favor of Markov–switching specifications.

Figures 6 and 7 plot the filtered and smoothed probabilities of the low–growth state for GDP and CP, respectively.[10] The filtered probabilities, $\Pr(S_t|\psi_t)$, denote the conditional probability that the state at date t is S_t, which is conditional upon the values of ψ observed through date t. The smoothed probabilities, $\Pr(S_t|\psi_T)$, on the other hand denote the conditional probability based on the data available from the whole sample at the future date T. The shaded areas are the contraction periods determined by the CEPD. For reference purposes, the recession periods identified by the CEPD are reported in the first column of Table 3 while the low growth regimes identified by GDP and CP are reported in the second and third columns of Table 3. Several observations may be extracted from those figures and Table 3. First, if we measure the business cycle only in terms of GDP, the inferred probabilities are not strongly consistent with the CEPD dating of the business cycle chronologies for the first and second recessions, but they successfully capture the

[10] We did not show the probability plots of IFIX and EX for space considerations. Detailed information is available from the authors upon request.

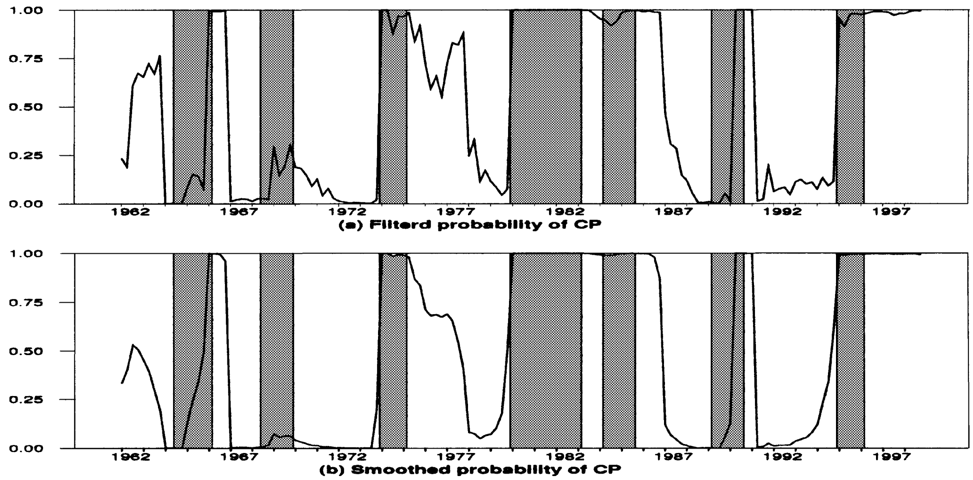

Figure 7 Filtered and Smoothed Probabilities of CP. The Shaded Areas Are the Contraction Periods Determined by the CEPD

last five recession periods, i.e., 1974–1975, 1980–1983, 1984–1985, 1989–1990 and 1995–1996, published by the CEPD. However, while the business chronologies cannot be dichotomized very well for the post–1990 period, these results are consistent with the empirical work of Lin and Chen (1999), Chen and Lin (2000) and Huang (1999). Second, although the business chronologies produced in relation to CP are similar to those in the case of GDP, they also miss the first two contraction periods and capture the last five business cycle chronologies identified by the CEPD. Significantly, the filtered and smoothed probabilities in the case of CP may dichotomize the expansion and contraction states better than GDP for the post–1990 period. Third, the synchronization in turning times in relation to both GDP and CP provides the impetus to the development of factor structure, as noted by Diebold and Rudebusch (1996), thereby improving the poor performance of the business chronologies for the post–1990 period.

Before we proceed to the multivariate factor models analysis, it is of interest to compare our univariate results with those of Huang, Kuan and Lin (1998) and Huang (1999), two recent empirical studies employing Markov–switching models to explore features of Taiwan's business cycles. The fourth and fifth columns of Table

Table 3 Business Cycle Chronology in Taiwan

CEPD (61:Q1–98:Q4) Peak/Trough	GDP (61:Q1–98:Q4) Peak/Trough	CP (61:Q1–98:Q4) Peak/Trough	Huang et al. (1998) (61:Q1–95:Q3) Trough	Huang (1999) (61:Q1–96:Q4) Peak/Trough
	62:Q1/62:Q4	62:Q3/62:Q4		63:Q2/63:Q4
64:Q3/66:Q1	65:Q4/66:Q3	66:Q1/66:Q4	66:Q1	64:Q3/66:Q2
68:Q3/69:Q4	68:Q1/68:Q2			68:Q1/70:Q1
74:Q1/75:Q1	74:Q1/75:Q1	74:Q1/77:Q3	75:Q1	74:Q1/75:Q1
				76:Q2/77:Q1
	77:Q3			77:Q3
80:Q1/83:Q1	78:Q4/83:Q4	80:Q1/NA	82:Q2	79:Q2/NA
			83:Q4	
84:Q2/85:Q3	84:Q2/85:Q4	NA/86:Q4	85:Q3	NA/85:Q4
89:Q2/90:Q3	88:Q1/NA	90:Q2/91:Q1	90:Q4	87:Q4/NA
95:Q1/96:Q1	NA/98:Q4	94:Q4/98:Q4		NA/96:Q4

Note: NA = not available.

3 summarize the business chronologies identified by Huang et al. (1998) and Huang (1999). There are several differences between our models and those presented in Huang, Kuan and Lin (1998) and Huang (1999). First, our paper is consistent with Huang (1999) in which all empirical models are of the univariate Markov–type with mean–switching, while Huang et al. (1998) uses a univariate Markov–switching model allowing only the intercept to switch from one state to another. Second, our paper and that of Huang (1999) both use seasonally–unadjusted real GDP as a summative measure of the business cycle, while Huang, Kuan and Lin (1998) use seasonally–unadjusted GNP to identify turning points in Taiwan. Third, Huang, Kuan and Lin (1998) and Huang (1999) employ the MLE method to implement the estimation, while we extend the model to the state-space representation and apply Kim's (1994) algorithm to estimate our models. Fourth, the empirical sample periods in each of three papers are, of course, different from each other. Since our empirical models are the same as those of Huang (1999), it is obvious from Table

3 that business chronologies identified in our paper are more consistent with those of Huang (1999). However, the univariate Markov–switching models used in the three papers all share a shortcoming: they did not identify turning points that were consistent with the dates identified by the CEPD for the post–1990 period.

3.3 Estimates of Multivariate Markov–Switching Factor Model

The MS(s)-DF(q, k) model is a synthesis of the dynamic factor model, developed by Stock and Watson (1989, 1991, 1993), and the Markov–switching model proposed by Hamilton (1989). The purpose of this model is to capture two defining characteristics of the business cycle: namely, co–movements among economic variables through the cycle and nonlinearity in terms of the evolution of the business cycle. We estimate a vector of macroeconomic variables and these are standardized by subtracting the sample mean from each variable and dividing the result by its standard deviation. It should be noted that the model does not assume that the series are cointegrated. Johansen's (1988) cointegration test, trace and λ_{max} statistic, is adopted to justify the evidence of a no cointegration relationship among the variables. The test results are included in Table 4, and it is easy to conclude that no cointegration exists among these variables. However, we also apply Stock and Watson's (1988) test statistic, q_f, to detect the number of stochastic common trends among the variables. We cannot reject one stochastic common trend among them, which implies that there is one cointegration relationship. To account for the possibility that GDP and CP may be cointegrated, we include an additional 2 lags of n_t in the equations for GDP and CP. Moreover, we do not restrict the factor so as to extract only the contemporaneous information about the state of the economy contained in the three series. Investment (IFIX) and exports (EX) may sometimes lead the business cycle and contain further information regarding the evolution of the economy. This leading behavior could be reflected in the behavior of the common factor.

We divide the empirical models into three groups, i.e., {GDP, CP and IFIX} as group one and denoted as MS–DF1, {GDP, CP and EX} and {GDP, IFIX and EX}, and groups 2 and 3 denoted as MS–DF2 and MS–DF3, respectively. Different

Table 4 Johansen's Cointegration Test

r	$p-r$	trace	λ_{max}	trace (95%)	λ_{max} (95%)
0	3	15.85	9.09	29.68	20.97
1	2	6.75	5.96	15.41	14.07
2	1	0.79	0.79	3.76	3.76
0	3	21.38	14.13	29.68	20.97
1	2	7.25	6.28	15.41	14.07
2	1	0.97	0.97	3.76	3.76
0	3	26.43	15.50	29.68	20.97
1	2	10.93	7.78	15.41	14.07
2	1	3.15	3.15	3.76	3.76

Note: The variables in the first block are GDP, CP and IFIX. Those in the second block are GDP, CP and EX. Those in the third block are GDP, IFIX and EX.

idiosyncratic components of the three groups, which appears in the form of either AR(1) or AR(2), are estimated. The final chosen models are based on the BIC and turning point criteria. According to the BIC criteria report in Table 5, the order of idiosyncratic components for the three chosen empirical models are 2, 1 and 1, respectively.

The first column of Table 5 reports the approximate maximum likelihood estimates (MLE) of the multivariate dynamic Markov–switching factor model of group one, {GDP, CP and IFIX}, with the AR(2) common factor and restricted VAR(2) idiosyncratic noise terms, denoted by MS(2)–DF1$(2,2)$. The second and the third columns report similar estimates of {GDP, CP and EX} and {GDP, IFIX and EX}, which are denoted by MS(2)–DF2$(2,1)$ and MS(2)–DF3$(2,1)$, respectively. Since the proposed MLE is an approximation, the numbers in parentheses represent the corresponding approximate standard errors. The estimate of the contractionary state (β_0) is significantly negative and the expansion state ($\beta_0+\beta_1$) is significantly positive as is expected.[11] Table 6 summarizes the business chronologies identified

[11] Since a satisfactory asymptotic distribution of the LR statistic is not available for the multivariate case, we take it for granted that our multivariate Markov–switching factor model is a 2–state economy, i.e., consisting of expansion and contraction phases, and we do not perform an explicit test for nonlinearity.

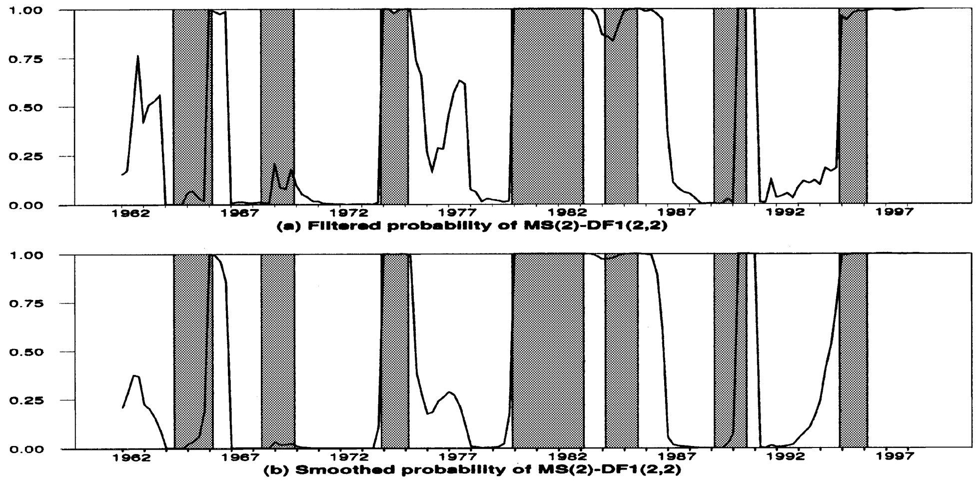

Figure 8 Filtered and Smoothed Probabilities of MS(2)–DF1(2,2). The Shaded Areas Are the Contraction Periods Determined by the CEPD

by the smoothed probabilities for the various multivariate models. Figures 8–10 are their corresponding plots of the filtered and smoothed probabilities for the contraction phases. Using the criterion proposed by Hamilton (1989) and Hamilton and Perez–Quiros (1996), we divide the samples with $\Pr(S_t = 1|\psi_T) \geq 0.5$ for the high–growth state, and with $\Pr(S_t = 0|\psi_T) \geq 0.5$ for the low–growth state. We make the following observations from Figures 8–10 and Table 6. First, the three multivariate Markov–switching factor models all miss the first two business cycle chronologies, i.e., 1964:Q3–1966:Q1 and 1968:Q3–1969:Q4, which are identified by the CEPD through the estimated samples. This result of missing the second recession period is consistent with Lin and Huang (1993) and Huang, Kuan and Lin (1998). The turning points identified by the CEPD are mainly based on the coincident index (CI). If we trace the history of the CI, the variation of the CI in the periods of 1968:Q3 to 1969:Q4 are relatively smooth and the difference between the peak and trough of this cycle is smaller than in the case of the other business cycles. This is the reason why our empirical models cannot classify this period as a business cycle in Taiwan. Second, all multivariate empirical models are able to capture the last five business cycles identified by the CEPD as univariate cases. Third, the MS(2)–DF2(2, 1) and MS(2)–DF3 (2, 1) models still cannot successfully classify the

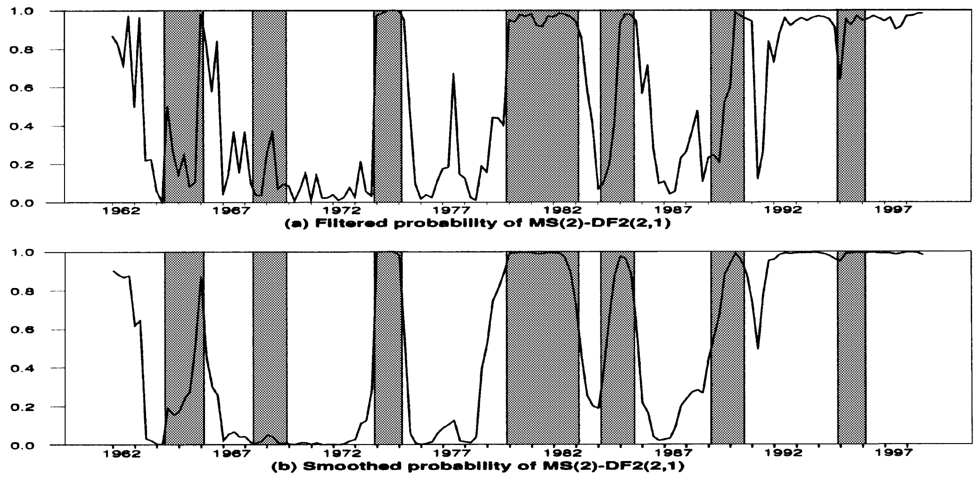

Figure 9 Filtered and Smoothed Probabilities of MS(2)–DF2(2,1). The Shaded Areas Are the Contraction Periods Determined by the CEPD

business chronologies for the post–1990 period. However, the MS(2)-DF1(2, 2) model in terms of the series {GDP, CP and IFIX} successfully identifies the turning points for the post–1990 period among all of the multivariate Markov–switching factor models.

The expected duration of the expansion and contraction are calculated using the formulae $(1-p)^{-1}$ and $(1-q)^{-1}$, respectively. The outstanding difference between the univariate model and the multivariate model is evident from the comparison of $p = \Pr(S_t = 1|S_{t-1} = 1)$. It should be remembered that $S_t = 1$ denotes the expansion state of the economy. The expansion period in the univariate GDP model is only about six quarters, while in the multivariate model it is about fourteen quarters. The fourteen expected expansion quarters in the multivariate models are equal to the periods identified by the CEPD.

3.4 Forecasting Performance

The in–sample and out–sample forecasting performance of univariate and multivariate models, following Hamilton and Perez–Quiros (1996), is based on the turning point (TP) criterion defined as follows:

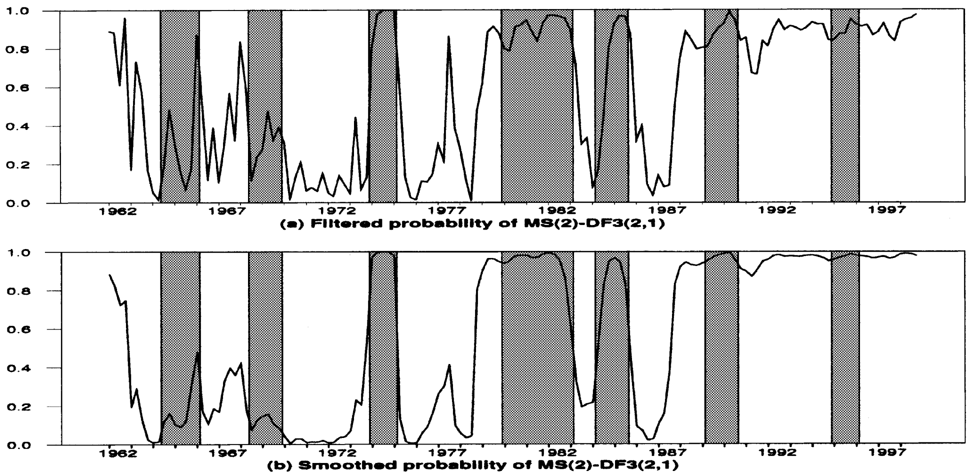

Figure 10 Filtered and Smoothed Probabilities of MS(2)–DF3(2,1). The Shaded Areas Are the Contraction Periods Determined by the CEPD.

$$\text{TP} = K^{-1} \sum_{t=1}^{K} \{\text{prob}(s_t = 0|\psi_T) - d_t\}^2 \tag{12}$$

where $d_t = 1$ if dated as a period of the CEPD–defined contraction. [12] We used the period 1963:Q1 to 1998:Q4 which amounts to 148 observations to calculate the in–sample TP. The calculation of the out–of–sample TP was as follows. The parameters were estimated using data from 1961:Q1 up to 1986:Q4, and the in–sample estimates were then used to generate out–of–sample forecasts of the filtered probabilities, which implies that the last $K = 48$ observations are reserved for the forecasting evaluation. However, the filtered probabilities rather than the smoothed probabilities are adopted for calculating the out–of–sample TP because obtaining the latter is problematical due to the nonlinearity of the Markov–switching factor model in out–of–sample estimation. As noted by Kim and Nelson (1999), "the smoothing algorithm for a state–space model with Markov switching is not as straightforward

[12] In fact, the turning point criterion defined in equation (12) was first proposed by Diebold and Rudebusch (1989) and alternatively named the quadratic probability score (QPS). Diebold and Rudebusch (1989) proposed the QPS as a measure of correspondence between turning point probabilities and actual turning points. By contrast, Filardo (1994) and Hamilton and Perez–Quiros (1996) used it in connection with the actual NBER phase dates and the model–generated regime probabilities for each data point in the series.

Table 5 Estimates of Multivariate Markov-Switching Factor Models

Parameter	MS(2)-DF1(2,2)	MS(2)-DF2(2,1)	MS(2)-DF3(2,1)
ϕ_1	−0.063 (0.127)	0.198 (0.117)	0.857 (0.178)
ϕ_2	−0.001 (0.004)	−0.010 (0.012)	−0.184 (0.076)
ϕ_{11}	0.809 (0.104)	0.671 (0.060)	0.621 (0.168)
ϕ_{12}	−0.066 (0.089)		
ϕ_{21}	1.227 (0.558)	0.863 (0.060)	0.621 (0.067)
ϕ_{22}	−0.349 (0.522)		
ϕ_{31}	0.521 (0.086)	0.372 (0.074)	−0.137 (0.152)
ϕ_{32}	0.126 (0.084)		
σ_1	0.571 (0.040)	0.536 (0.032)	0.319 (0.097)
σ_2	0.268 (0.168)	0.351 (0.051)	0.768 (0.045)
σ_3	0.752 (0.044)	0.803 (0.046)	0.601 (0.069)
γ_1	0.086 (0.029)	−0.014 (0.014)	0.412 (0.071)
γ_2	0.244 (0.076)	−0.017 (0.013)	0.061 (0.059)
γ_3	0.065 (0.039)	−0.086 (0.034)	0.343 (0.054)
γ_{11}	0.029 (0.028)	−0.029 (0.021)	−0.078 (0.078)
γ_{12}	0.003 (0.024)	−0.083 (0.032)	0.079 (0.054)
γ_{21}	0.001 (0.023)	0.051 (0.022)	0.002 (0.043)
γ_{21}	−0.004 (0.024)	−0.192 (0.066)	−0.021 (0.057)
β_0	−3.574 (1.862)	−5.114 (2.158)	−0.647 (0.286)
β_1	7.021 (2.381)	9.480 (3.751)	1.454 (0.544)
p	0.928 (0.032)	0.925 (0.032)	0.892 (0.086)
q	0.931 (0.035)	0.945 (0.025)	0.922 (0.054)
$(1-p)^{-1}$	13.889	13.333	9.259
$(1-q)^{-1}$	14.493	18.182	12.821
# of parameters	22	19	19
Log-Likelihood	−10.621	−18.142	−48.369
BIC	−34.579	−38.759	−68.986

Note: The numbers in parentheses are approximate standard errors. The BIC for models of MS(2)–DF1(2,1), MS(2)–DF2(2,2) and MS(2)–DF3(2,2) are −35.579, −45.174 and −70.588, respectively.

Table 6 Business Cycle Chronology in Taiwan

CEPD (61:Q1–98:Q4) Peak/Trough	MS(2)-DF1(2,2) (61:Q1–98:Q4) Peak/Trough	MS(2)-DF2(2,1) (61:Q1–98:Q4) Peak/Trough	MS(2)-DF3(2,1) (61:Q1–98:Q4) Peak/Trough
		62:Q1/63:Q2	62:Q1/62:Q4
64:Q3/66:Q1	66:Q1/66:Q4	65:Q4/66:Q1	
68:Q3/69:Q4			
74:Q1/75:Q1	74:Q1/75:Q2	74:Q1/75:Q2	74:Q1/75:Q1
80:Q1/83:Q1	80:Q3/NA	79:Q1/83:Q1	78:Q4/83:Q2
84:Q2/85:Q3	NA/86:Q4	84:Q3/85:Q4	84:Q2/85:Q3
89:Q2/90:Q3	90:Q2/91:Q1	89:Q2/91:Q1	87:Q4/NA
95:Q1/96:Q1	94:Q3/98:Q4	91:Q3/98:Q4	NA/98:Q4

Note: NA=not available.

as that for a linear state–space model. The filtered estimates of the unobserved components involve approximation. Such approximations are necessary to make the Kalman filter operable. In order to get smoothed estimates, one needs additional approximations as Kim (1994) has shown." The third and fourth columns of Table 7 summarize the in–sample and one–period–ahead out–of–sample forecasting performance, respectively. For the univariate case, the in–sample TP for GDP, CP and IFIX are 0.341, 0.308 and 0.324, respectively. It is apparent that CP has the best fit among the univariate Markov–switching models. Based on our observations of the out–of–sample TP forecasting, CP also does a better job in forecasting than GDP and IFIX, whereas for the multivariate case, the MS(2)–DF1(2, 2) model with variables {GDP, CP and IFIX} performs the best among all of the multivariate models in terms of in–sample and out–sample forecasting precision. Figure 11 summarizes the corresponding out–of–sample filtered probabilities for the recession regime. These plots also support the TP criterion reported in Table 7.

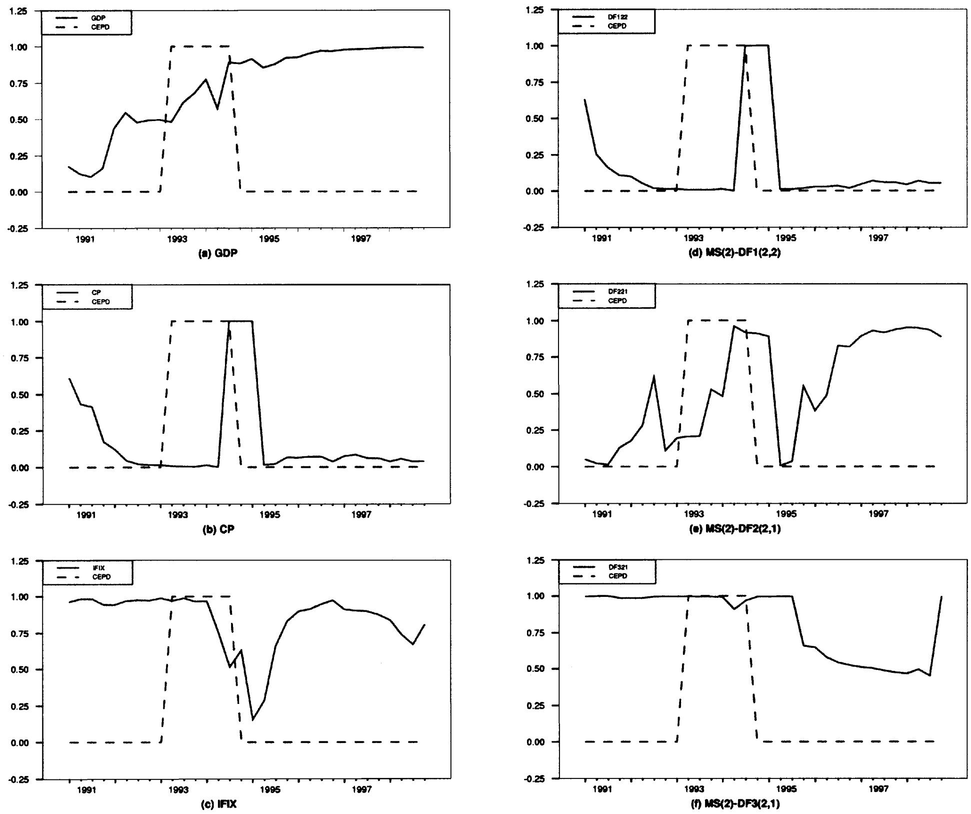

Figure 11 The Out-of-Sample Filtered Probability for the Recession Regime of Empirical Models

4. CONCLUSIONS

This paper builds upon the ideas set forth by Diebold and Rudebusch (1996) and estimates a multivariate dynamic Markov-switching factor model for a vector of macroeconomic variables. The approach adopted captures both the idea of the business cycle being depicted by co-movements in several macroeconomic variables and the asymmetric nature of business cycle phases. The univariate Markov-switching models are also estimated for reference purpose. We transform the empirical models into a state-space representation, and adopt Kim's (1994) algorithm to implement the estimation. The empirical results suggest that the business cycle chronologies

Table 7 Forecasting Performance

Models	Log-Lik.	BIC	TP (In-sample)	TP (Out-sample)
GDP	−299.907	−307.503	0.341	0.409
CP	−251.262	−257.773	0.308	0.402
IFIX	−482.262	−489.858	0.324	0.418
MS(2)-DF1(2,2)	−10.621	−34.493	0.284	0.401
MS(2)-DF2(2,1)	−18.142	−38.759	0.298	0.478
MS(2)-DF3(2,1)	−48.369	−68.986	0.336	0.540

identified by the multivariate Markov–switching factor model in terms of GDP, consumption and investment are more consistent with the CEPD–defined chronologies than they are in terms of the univariate Markov–switching models, especially for the post–1990 period.

The method we employ in this paper is the approximate maximum–likelihood estimation method used by Kim (1994) to estimate the nonlinear dynamic factor model of the business cycle. As noted by Kim and Nelson (1998), although the maximum–likelihood estimation based on approximations to the Kalman filter is straightforward to implement, it is difficult to judge the effects of the approximations on the parameter estimates and on inferences pertaining to the unobserved common component n_t and the unobserved states S_t. They suggested using Gibbs sampler approach to estimate the empirical models. Furthermore, the business cycle chronologies identified by the CEPD are based on the coincident index. The coincident indicator is composed of six important components, i.e., the industrial production index, manufacturing sales, average monthly wages and salaries, bank clearing, the quantum of domestic traffic and the manufacturing production index. For the empirical investigation of each component of the coincident index in Taiwan, the reader may refer to Lin and Huang (1993). However, their empirical model did not take care of the nonlinearity feature of the business cycle. Further research on this issue involving applying the Gibbs sampler approach remains a high priority on our research agenda for the near future.

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Appendix : Estimation and Algorithm

In this appendix, we discuss the estimation of the model based on Kim's (1994) approximate MLE. Basically, Kim's algorithm is a synthesis of Hamilton's filter and the Kalman filter. For example, the multivariate dynamic Markov-switching factor model with $\mathbf{y}_t = [y_{1t}\ y_{2t}\ y_{3t}]'$, and AR(2) common factor and AR(2) idiosyncratic components may be depicted as follows:

$$\mathbf{y}_t = \mathbf{H}_t \boldsymbol{\xi}_t \tag{13}$$

$$\boldsymbol{\xi}_t = \mathbf{F}_t \boldsymbol{\xi}_{t-1} + \boldsymbol{\beta}_{S_t} + \mathbf{u}_t \tag{14}$$

$$\mathbf{H}_t = \begin{bmatrix} \gamma_1 & \gamma_{11} & \gamma_{12} & 1 & 0 & 0 & 0 & 0 & 0 \\ \gamma_2 & \gamma_{21} & \gamma_{22} & 0 & 0 & 1 & 0 & 0 & 0 \\ \gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{F}_t = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_{11} & \phi_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi_{21} & \phi_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi_{31} & \phi_{32} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\boldsymbol{\xi}_t = \begin{bmatrix} n_t \\ n_{t-1} \\ n_{t-2} \\ z_{1,t} \\ z_{1,t-1} \\ z_{2,t} \\ z_{2,t-1} \\ z_{3,t} \\ z_{3,t-1} \end{bmatrix}, \ \boldsymbol{\beta}_{S_t} = \begin{bmatrix} \beta_{S_t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{u}_t = \begin{bmatrix} \eta_t \\ 0 \\ 0 \\ \varepsilon_{1t} \\ 0 \\ \varepsilon_{2t} \\ 0 \\ \varepsilon_{3t} \\ 0 \end{bmatrix} \ \mathbf{Q}_t = \begin{bmatrix} \sigma_\eta^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\varepsilon_1}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\varepsilon_2}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\varepsilon_3}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

where $\mathbf{Q} = \mathrm{E}(\mathbf{u}_t \mathbf{u}_t')$. Given a realization of the state variables at time t and $t-1$ ($S_t = j$ and $S_{t-1} = i$, where $i,\ j$ =0 or 1) and using the notation $Z_{t|t-1}^{(i,j)}$ to denote the variable Z as being conditional upon the information available up to $t-1$ and the realized states j and i, the Kalman filter can be represented as follows.

$$\boldsymbol{\xi}_{t|t-1}^{(i,j)} = \mathbf{F}_t \boldsymbol{\xi}_{t-1|t-1}^{(i)} + \boldsymbol{\beta}_{S_t}^{(j)} \tag{15}$$

$$\mathbf{P}^{(i,j)}_{t|t-1} = \mathbf{F}_t \mathbf{P}^{(i)}_{t-1|t-1} \mathbf{F}'_t + \mathbf{Q}_t \tag{16}$$

$$\boldsymbol{\xi}^{(i,j)}_{t|t} = \boldsymbol{\xi}^{(i,j)}_{t|t-1} + \mathbf{K}^{(i,j)}_t \boldsymbol{\eta}^{(i,j)}_{t|t-1} \tag{17}$$

$$\mathbf{P}^{(i,j)}_{t|t} = (\mathbf{I} - \mathbf{K}^{(i,j)}_t \mathbf{H}_t) \mathbf{P}^{(i,j)}_{t|t-1} \tag{18}$$

$$\boldsymbol{\eta}^{(i,j)}_{t|t-1} = \mathbf{y}_t - \mathbf{H}_t \boldsymbol{\xi}^{(i,j)}_{t|t-1} \tag{19}$$

$$\mathbf{W}^{(i,j)}_{t|t-1} = \mathbf{H}_t \mathbf{P}^{(i,j)}_{t|t-1} \mathbf{H}'_t \tag{20}$$

$$\mathbf{K}^{(i,j)}_t = \mathbf{P}^{(i,j)}_{t|t-1} \mathbf{H}'_t (\mathbf{W}^{(i,j)}_{t|t-1})^{-1} \tag{21}$$

where equations (15) and (16) are the prediction formulae, equations (17) and (18) are the updating formulae and equation (21) is the Kalman gain. $\boldsymbol{\eta}^{(i,j)}_{t|t-1}$ is the conditional forecast error of $\mathbf{y}_t$ based on information up to $t-1$, and $\mathbf{W}^{(i,j)}_{t|t-1}$ is the conditional variance of forecast error $\boldsymbol{\eta}^{(i,j)}_{t|t-1}$. As noted by Harrison and Stevens (1976), each iteration of the above Kalman filtering produces a 2–fold increase in the number of cases to consider. Kim (1994) provided a fast approximation algorithm applicable to this problem. The idea is to collapse the dimension of the (2×2) posteriors ($\boldsymbol{\xi}^{(i,j)}_{t|t}$ and $\mathbf{P}^{(i,j)}_{t|t}$) to two posteriors ($\boldsymbol{\xi}^{(j)}_{t|t}$ and $\mathbf{P}^{(j)}_{t|t}$) by taking weighted averages over states at $t-1$. That is,

$$\boldsymbol{\xi}^{(j)}_{t|t} = \frac{\sum_{S_{t-1}=0}^{1} \Pr[S_t = j,\ S_{t-1} = i|\psi_t] \times \boldsymbol{\xi}^{(i,j)}_{t|t}}{\Pr[S_t = j|\psi_t]} \tag{22}$$

$$\mathbf{P}^{(j)}_{t|t} = \frac{\sum_{S_{t-1}=0}^{1} \Pr[S_t = j,\ S_{t-1} = i|\psi_t] \times \{\mathbf{P}^{(i,j)}_{t|t} + (\boldsymbol{\xi}^{(j)}_{t|t} - \boldsymbol{\xi}^{(i,j)}_{t|t})(\boldsymbol{\xi}^{(j)}_{t|t} - \boldsymbol{\xi}^{(i,j)}_{t|t})'\}}{\Pr[S_t = j|\psi_t]} \tag{23}$$

where ψ_t refers to information available at time t. Following Hamilton (1989, 1990), the filter can be obtained using Bayes's theorem.

$$\Pr[S_t = j,\ S_{t-1} = i|\psi_t] = \frac{\Pr[\mathbf{y}_t, S_t = j, S_{t-1} = i|\psi_{t-1}]}{\Pr[\mathbf{y}_t|\psi_{t-1}]}$$

$$= \frac{f[\mathbf{y}_t|S_t=j,\ S_{t-1}=i,\ \psi_{t-1}] \times \Pr[S_t=j,\ S_{t-1}=i|\psi_{t-1}]}{\Pr[\mathbf{y}_t|\psi_{t-1}]} \tag{24}$$

where

$$f[\mathbf{y}_t|S_t=j,\ S_{t-1}=i,\ \psi_{t-1}] = (2\pi)^{-N/2}|\mathbf{W}^{(i,j)}_{t|t-1}|^{-1/2} \times \exp\{-\frac{1}{2}\boldsymbol{\eta}^{(i,j)\prime}_{t|t-1}(\mathbf{W}^{(i,j)}_{t|t-1})^{-1}\boldsymbol{\eta}^{(i,j)}_{t|t-1}\} \tag{25}$$

The smoothed probabilities, $p(S_t|\psi_T)$, on the other hand are the conditional probabilities which are based on data available through the whole sample at future date T, and amount to

$$\Pr[S_{t+1}=k, S_t=j|\psi_T] \approx \frac{\Pr[S_{t+1}=k|\psi_T]\times\Pr[S_t=j|\psi_t]\times\Pr[S_{t+1}=k|S_t=j]}{\Pr[S_{t+1}=k|\psi_t]} \tag{26}$$

$$\Pr[S_t=j|\psi_T] = \sum_{S_{t+1}=0}^{1}\Pr[S_{t+1}=k,\ S_t=j|\psi_T] \tag{27}$$

The approximate sample conditional log-likelihood is

$$\text{LL} = \ln f(\mathbf{y}_T, \mathbf{y}_{T-1}, \cdots|\psi_0) = \sum_{t=1}^{T}\ln f(\mathbf{y}_t|\psi_{t-1}) \tag{28}$$

The approximate ML estimates of the model can be obtained by maximizing the log–likelihood with respect to the unknown parameters.

臺灣景氣循環轉折點之認定：多變量動態馬可夫轉換單因子模型之應濾

陳仕偉
助理教授
東海大學經濟系

林金龍
研究員
中央研究院經濟研究所

摘　要

本文應用多變量動態馬可夫轉換單因子模型，對台灣重要總體經濟變數如國內生產毛濾濾濾濾濾濾濾濾濾濾濾濾濾濾濾，濾濾台灣經濟濾濾濾濾轉濾濾濾濾濾濾濾濾多變量動態馬可夫轉換單因子模型濾濾濾濾濾濾濾濾濾濾濾濾濾對濾濾濾濾濾，濾濾濾濾濾濾濾濾濾濾濾重要濾濾濾濾重要總體經濟變數如濾濾濾濾濾濾濾濾濾濾濾濾濾動濾國內生產毛濾濾動濾濾變動濾濾濾濾，濾濾單變量馬可夫轉換模型濾濾濾濾濾濾濾濾濾濾濾濾多變量動態馬可夫轉換單因子模型濾濾單變量馬可夫轉換模型濾濾濾濾台灣濾濾濾濾轉濾濾濾濾濾濾濾濾，濾濾濾濾 1990 濾濾濾濾濾

濾濾濾濾馬可夫轉換單因子模型濾濾態濾濾模型濾濾濾濾濾濾濾濾濾濾